

**THE CAPUTO DERIVATIVE, FELLER SEMIGROUPS,
AND THE FRACTIONAL POWER OF
THE FIRST ORDER DERIVATIVE ON $C_\infty(\mathbb{R}_0^+)$**

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*Dedicated to Professor Francesco Mainardi
on occasion of his 60th birthday*

Abstract

We study the fractional derivatives of order $\alpha \in (0, 1)$ of Riemann-Liouville form D_R^α and of Caputo form D_C^α on the set $C_\infty(\mathbb{R}_0^+)$ of continuous functions on $\mathbb{R}_0^+ = [0, \infty)$ vanishing at infinity. We construct Feller semigroups generated by $-D_C^\alpha$ with the homogeneous Dirichlet and Neumann boundary condition respectively. From the expressions obtained we deduce that the Caputo form $-D_C^\alpha$ is the fractional power of the first order derivative, when it is considered as a generator of a Feller semigroup on $C_\infty(\mathbb{R}_0^+)$.

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